Design Equations for Symmetric Microstrip DC Blocks

DARKO KAJFEZ, SENIOR MEMBER, IEEE, AND B. SARMA VIDULA, MEMBER, IEEE

***Abstract*—The design formulas are presented for achieving either a rippled or a maximally flat response of the dc blocks built with symmetrical coupled microstrip lines. The selection of characteristic impedances of the odd and even modes is facilitated by the use of a universal diagram containing the equicontours of the standing wave ratio and of the bandwidth. The deformation of the frequency response due to a difference in wavelengths of the odd and even modes is analyzed and the design procedure is adjusted accordingly.**

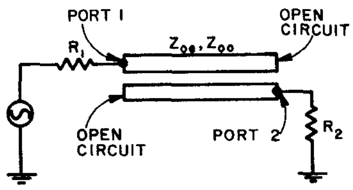

Fig. 1. Coupled transmission-line dc block.

I. Introduction

A SECTION of parallel coupled symmetric transmission line can be used to transmit certain range of microwave frequencies without attenuation, but acts as an open circuit at dc. La Combe and Cohen [1] have named such a device a dc block, and reported the experimental data showing a wide-band operation of the device designed on microstrip. Rizzoli [2] has approached the design of dc blocks from the TEM assumption, and presented design formulas and graphs for selected special cases. We intend to present the results of further investigation of the properties of the dc blocks. Our analysis is based on an exact expression of the scattering parameter S_{21} which fully takes into account the possibility that the propagation modes on coupled microstrip are of the quasi TEM type, and that the input and output impedances may be different from each other. It will be shown that the design procedure can be based on the equations for the pure TEM modes, and that the presence of the quasi TEM modes requires only a minor correction of the physical length of the coupled section. It will also be shown that there exist two different solutions both yielding a rippled response with the prescribed bandwidth and the prescribed standing wave ratio. The presentation is organized in such a way that either the values of Z_{oe} and Z_{oo} can be computed for a prescribed bandwidth and prescribed standing wave ratio or the attainable bandwidth and standing wave ratio may be read from the curves on the normalized Z_{oe}, Z_{oo} plane.

II. Frequency Response of the Transmission Coefficient

The dc block in Fig. 1 is inserted between a generator with internal resistance R_1 and load resistance R_2. The operating properties of this circuit can be best described by the scattering matrix transmission coefficient S_{21}. For the case that the even and the odd mode wavelengths differ from each other due to the presence of inhomogeneous dielectrics, the transmission coefficient is

$$S_{21}=\frac{2(Z_{oe}\operatorname{cosec}\theta_e - Z_{oo}\operatorname{cosec}\theta_o)}{\left\{\left[\sqrt{\frac{R_2}{R_1}}+\sqrt{\frac{R_1}{R_2}}\right](Z_{oe}\cot\theta_e + Z_{oo}\cot\theta_o) + j\left[\frac{Z_{oe}^2+Z_{oo}^2}{2\sqrt{R_1R_2}} - \frac{Z_{oe}Z_{oo}}{\sqrt{R_1R_2}}(\operatorname{cosec}\theta_e\operatorname{cosec}\theta_o + \cot\theta_e\cot\theta_o) + 2\sqrt{R_1R_2}\right]\right\}}. \tag{1}$$

The above equation has been derived from the formulas given by Zysman and Johnson [3] by performing the transformation from the chain matrix to scattering matrix [4]. The electric lengths of the even and odd mode (in radians) are

$$\theta_e = kl\sqrt{\epsilon_{re}} \qquad \theta_o = kl\sqrt{\epsilon_{ro}} \tag{2}$$

with k being the free-space propagation constant

$$k=\omega\sqrt{\mu_o\epsilon_o}\,. \tag{3}$$

Symbols ϵ_{ro} and ϵ_{re} denote the relative dielectric constants of the odd and even modes. The coupled transmission line has length l, and it is symmetric so that the modes of propagation are strictly odd and strictly even, described by the characteristic impedances Z_{oo} and Z_{oe}.

It is convenient to introduce the normalized odd and even mode impedances

$$z_e=\frac{Z_{oe}}{\sqrt{R_1R_2}} \qquad z_o=\frac{Z_{oo}}{\sqrt{R_1R_2}} \tag{4}$$

Manuscript received December 31, 1979; revised April 15, 1980.

D. Kajfez is with the Department of Electrical Engineering, University of Mississippi, University, MS 38677.
B. S. Vidula is with MATV Engineering, Channel Master, Div. of AVNET, Inc., Ellenville, NY 12428.

0018-9480/80/0900-0974$00.75 ©1980 IEEE

and the impedance transformation parameter

$$\kappa^2 = \frac{(R_1+R_2)^2}{4R_1R_2}. \tag{5}$$

The frequency dependence will be described in terms of variables Ω_e and Ω_o

$$\Omega_e = \cot\theta_e \qquad \Omega_o = \cot\theta_o. \tag{6}$$

The quantity of interest in the design of the dc block is the squared amplitude of the transmission coefficient, the transmission loss

$$|S_{21}|^2 = \frac{4\left(z_e\sqrt{1+\Omega_e^2} - z_o\sqrt{1+\Omega_o^2}\right)^2}{\left(4\kappa^2(z_e\Omega_e + z_o\Omega_o)^2 + \left\{\frac{1}{2}(z_e^2+z_o^2+4) - z_ez_o\left[\Omega_e\Omega_o + \sqrt{(1+\Omega_e^2)(1+\Omega_o^2)}\right]\right\}\right)^2}. \tag{7}$$

From this expression it is possible to study how the difference in the wavelengths of the odd and even modes influences the overall response as a function of frequency. This will be done in Section VI. In order to understand the basic mechanism of the frequency dependence, (7) will be first simplified for the special case of pure TEM propagation, when the two modes have the same wavelength

$$\theta_o = \theta_e = \theta. \tag{8}$$

Then, the expression for the transmission loss takes the following form:

$$|S_{21}|^2 = \frac{1+\Omega^2}{a_o + a_2\Omega^2 + a_4\Omega^4}. \tag{9}$$

The coefficients of the polynomial in the denominator are

$$a_o = \left[\frac{(z_e-z_o)^2+4}{4(z_e-z_o)}\right]^2 \tag{10}$$

$$a_2 = \frac{4\kappa^2(z_e-z_o)^2 - 2z_ez_o\left[(z_e-z_o)^2+4-8\kappa^2\right]}{4(z_e-z_o)^2} \tag{11}$$

$$a_4 = \left(\frac{z_ez_o}{z_e-z_o}\right)^2. \tag{12}$$

As seen in (9) $|S_{21}|^2$ is a ratio of a linear and another quadratic expression in terms of Ω^2. Naturally, the idea occurs to choose the coefficients in such a manner that some Chebyshev-like or a Butterworth-like response is obtained. This can indeed be achieved if appropriate care is taken in two inconveniences: first, (10) to (12) are nonlinear in z_e and z_o, and second, there are three coefficients: a_0, a_2, and a_4, but only two really independent variables: z_e and z_o. In most applications, κ^2 is namely equal to unity.

The transmission loss is a symmetric function of frequency variable Ω. By a proper choice of coefficients a_0, a_2, and a_4, the frequency dependence may be shaped in the form of a ripple such as shown in Fig. 2. At the center frequency of operation, the frequency variable Ω is zero, and $|S_{21}|^2 = 1/a_0$. If the dc block is assumed to be lossless, the standing wave ratio S at the input port is related to a_0 as follows:

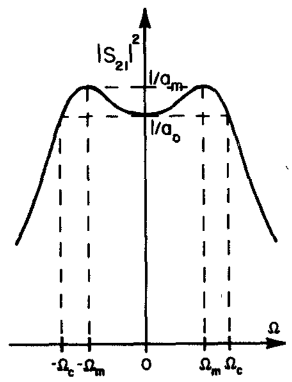

Fig. 2. Rippled response.

$$a_0 = \frac{(S+1)^2}{4S}. \tag{13}$$

The position of the two maxima in Fig. 2 may be obtained from the requirement that the first derivative of (9) with respect to Ω is zero. This requirement gives

$$\Omega_m^2 = -1 + \sqrt{1 + \frac{a_0 - a_2}{a_4}}. \tag{14}$$

The cutoff point Ω_c is defined as a point in which $|S_{21}|^2$ takes the same value $1/a_0$ as at the center frequency

$$\Omega_c^2 = \frac{a_0 - a_2}{a_4}. \tag{15}$$

The value of Ω_c specifies the frequency bandwidth as follows: If the lower edge frequency is f_1 and the upper edge frequency is f_2, then the relative bandwidth can be defined as

$$B_r = 2\frac{f_2 - f_1}{f_2 + f_1}. \tag{16}$$

The relationship with the variable Ω_c is then

$$\Omega_c = \cot\left[\frac{\pi}{2}\left(1 - \frac{B_r}{2}\right)\right]. \tag{17}$$

The second derivative of $|S_{21}|^2$ at $\Omega = 0$ is

$$\left.\frac{d^2|S_{21}|^2}{d\Omega^2}\right|_{\Omega=0} = \frac{2(a_0 - a_2)}{a_0^2}. \tag{18}$$

The condition for the existence of ripple is that (18) is positive. If (18) is made equal to zero, a maximally flat response is obtained (see Section. IV).

III. Design for Ripple Response

Coefficient a_0 determines the value of the response at the center frequency, as shown if Fig. 2. Since the input standing wave ratio S should not exceed certain prescribed value, a_0 is specified by (13). Solving (10) for $z_e - z_o$, the following two solutions are obtained:

$$z_{e1} - z_{o1} = 2\sqrt{S} \qquad z_{e2} - z_{o2} = \frac{2}{\sqrt{S}}. \tag{19}$$

Subscripts 1 and 2 have been used to denote solution one and solution two correspondingly. From (12) it follows that the two solutions are related as

$$z_{e1}z_{o1}=2\sqrt{S}\,\sqrt{a_{41}} \qquad z_{e2}z_{o2}=\frac{2\sqrt{a_{42}}}{\sqrt{S}}. \tag{20}$$

A second subscript has been added to a_4 to denote the possibility of two different choices for this coefficient, corresponding to either solution one or solution two.

An important design parameter is the bandwidth of the device which is related to the coefficients a_0, a_2, and a_4 by (15). Since a_0 is already fixed by (13), it is now possible to eliminate a_2 from (11) and (15) and find the values a_{41} and a_{42} which are to be used in (20).

It is noted that (11) contains an additional parameter κ^2, which does not appear in (10) or (12). The significance of this impedance transformation parameter may be a topic of a separate investigation [4], [5]. At present, it will be assumed that the source and the load impedances are equal to each other, so that κ^2 is unity. Then, the two positive solutions for $\sqrt{a_4}$ are found from (11) and (15) as follows:

$$\sqrt{a_{41}}=\frac{S-1}{2\sqrt{S}\,\Omega_c^2}\left[1+\sqrt{1+\Omega_c^2}\,\right] \tag{21}$$

$$\sqrt{a_{42}}=\frac{S-1}{2\sqrt{S}\,\Omega_c^2}\left[-1+\sqrt{1+\Omega_c^2}\,\right]. \tag{22}$$

These values are now substituted in (20), which together with (19) gives a system of two equations for the normalized even and odd mode impedances in terms of the prescribed standing wave ratio and bandwidth. Therefore, when $R_1=R_2$, the following design equations are obtained for solution one:

$$z_{e1}=\sqrt{S}\left[1+\sqrt{1+\frac{1+\sqrt{1+\Omega_c^2}}{\Omega_c^2}\left(1-\frac{1}{S}\right)}\,\right] \tag{23}$$

$$z_{o1}=\sqrt{S}\left[-1+\sqrt{1+\frac{1+\sqrt{1+\Omega_c^2}}{\Omega_c^2}\left(1-\frac{1}{S}\right)}\,\right]. \tag{24}$$

The design equations for solution two are

$$z_{e2}=\frac{1}{\sqrt{S}}\left[1+\sqrt{1+\frac{-1+\sqrt{1+\Omega_c^2}}{\Omega_c^2}(S-1)}\,\right] \tag{25}$$

$$z_{o2}=\frac{1}{\sqrt{S}}\left[-1+\sqrt{1+\frac{-1+\sqrt{1+\Omega_c^2}}{\Omega_c^2}(S-1)}\,\right]. \tag{26}$$

An interesting conclusion which can be reached from these formulas is the fact that any arbitrary combination of the chosen values for the bandwidth and the standing wave ratio can be achieved by real and positive values of z_e and z_o. Whether these values can be fabricated in a given microstrip configuration is a question which will be addressed later in Section V.

The magnitude of the scattering coefficient S_{21} of a passive two-port cannot be larger than unity. In Fig. 2, the maximum value of $|S_{21}|^2$ is denoted by $1/a_m$. For a ripple design a_m is bounded by

$$1\leqslant a_m<a_0. \tag{27}$$

It is of interest to find the values which a_m attains for the two designs named solutions one and two. When $\Omega=\Omega_m$, such as specified by (14), a_m can be computed from (9) as follows:

$$a_m=a_0-a_4\Omega_m^4. \tag{28}$$

For solution one, a_4 is specified by (21). It is then found that

$$a_{m1}=1 \tag{29}$$

for any selection of Ω_c and S. Therefore, when z_e and z_o are selected in accordance with (23) and (24), the resulting response $|S_{21}|^2$ always touches the value of unity at the two maxima shown in Fig. 2. On the other hand, solution two results in the following a_m:

$$a_{m2}=\frac{(S+1)^2}{4S}-\frac{(S-1)^2}{4S\Omega_c^4}\left[-1+\sqrt{1+\Omega_c^2}\,\right]^2. \tag{30}$$

This value never becomes unity, but is always smaller than a_0 in accordance with (27). For small and moderate bandwidths $\Omega_c<1$, and a_{m2} is only slightly different from a_0. In that case, the frequency response of solution two looks almost like a maximally flat response.

IV. Design for Maximally Flat Response

The frequency becomes maximally flat when the second derivative from (18) vanishes, i.e., when a_2 is equal to a_0. If a perfect impedance match is to be achieved at the center frequency, a_0 should be unity. Therefore, a maximally flat dc block, which is also reflectionless at the center frequency, requires the following choice of coefficients:

$$a_0=a_2=1. \tag{31}$$

Substituting (31) into (9), the following frequency response is obtained:

$$|S_{21}|^2=\frac{1+\Omega^2}{1+\Omega^2+a_4\Omega^4}. \tag{32}$$

Therefore, a_4 is the only parameter which can be selected to influence the bandwidth of the device.

Comparing Fig. 3 with Fig. 2, is seen that the bandwidth and the cutoff frequency of the maximally flat device must be defined in a somewhat different manner than in the case of the ripple response. Hence, the cutoff frequency Ω_c will be defined as the frequency at which the coefficient $|S_{21}|^2$ drops to the value $1/a_c$, where a_c is related to the input standing wave ratio S_c in the same

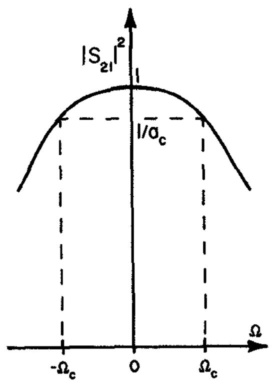

Fig. 3. Maximally flat response.

way as in (13)

$$a_c = \frac{(S_c+1)^2}{4S_c}. \tag{33}$$

The relationship between Ω_c and the relative bandwidth B_r remains the same as in (17).

When the requirement $a_0 = 1$ is substituted in (10), the solution for $z_e - z_o$ is found to be single valued

$$z_e - z_o = 2. \tag{34}$$

When this is used in (12) it follows

$$z_e z_o = 2\sqrt{a_4}. \tag{35}$$

It was found earlier that the maximally flat response requires a_2 to be unity. From (11), (34), and (35) it follows

$$a_2 = \kappa^2 + 2\sqrt{a_4}\,(\kappa^2 - 1). \tag{36}$$

When the impedance transforming parameter κ^2 is larger than unity (i.e., when $R_1 \neq R_2$), it is impossible to make a_2 equal to unity. Therefore, the maximally flat response can be achieved only when the input and output resistances are the same. In such a nontransforming case $\kappa^2 = 1$, and a_2 in (36) is identically equal to unity, irrespective of the value a_4.

At the cutoff frequency Ω_c the value of $|S_{21}|^2$ drops to $1/a_c$. The value of a_4 is then found from (32)

$$a_4 = \frac{(1+\Omega_c^2)(a_c - 1)}{\Omega_c^4} \tag{37}$$

where a_c is specified by (33). The above value of a_4 is now substituted in (35), which together with (34) gives a system of two equations for z_e and z_o. The design equations for the maximally flat response are then found to be

$$z_e = 1 + \sqrt{1 + \frac{S_c - 1}{\sqrt{S_c}} \cdot \frac{\sqrt{1+\Omega_c^2}}{\Omega_c^2}} \tag{38}$$

$$z_o = -1 + \sqrt{1 + \frac{S_c - 1}{\sqrt{S_c}} \cdot \frac{\sqrt{1+\Omega_c^2}}{\Omega_c^2}}\,. \tag{39}$$

It is seen that the real positive solutions will be obtained

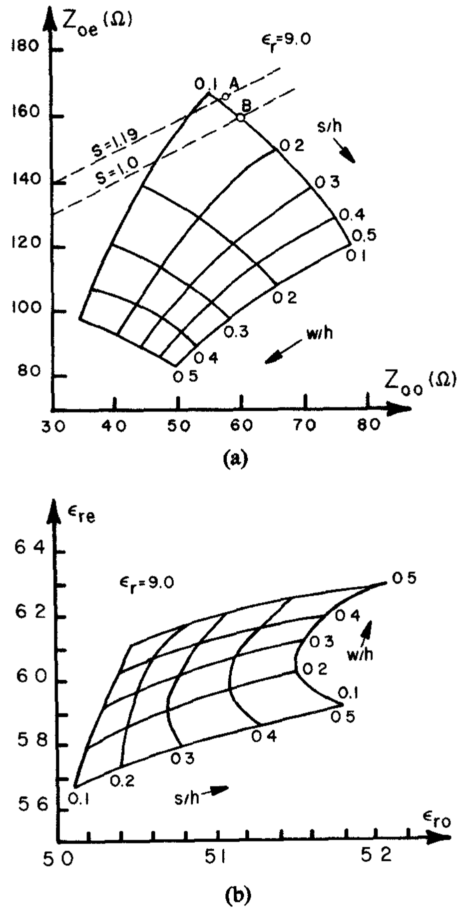

Fig. 4. (a) (Z_{oe}, Z_{oo}) plane for substrate $\epsilon_r = 9$. (b) $(\epsilon_{re}, \epsilon_{ro})$ plane for substrate $\epsilon_r = 9$.

for z_e and z_o with arbitrary choice of design parameters S and Ω_c.

When the coefficient a_4 is selected to be equal to 0.25, the microstrip dc block achieves the dimensions of a 3-dB directional coupler. This particular case has been analyzed by Ho [6], and has been later found by Rizzoli [2] as unfeasible in conventional microstrip technology. Our equation (37) offers more room for compromise between the bandwidth and the standing wave ratio.

V. Graphical Presentation on the (z_e, z_o) Plane

For a given substrate, the possible range of values Z_{oe} and Z_{oo} may be plotted in a diagram such as shown in Fig. 4(a), which has been introduced by Chambers [7]. The parameters in this diagram are the strip width w and the gap width s. The advantage of this representation is that one can easily spot the areas which are not well suited for fabrication, either because of narrow gap, or because of narrow conductor width. From the designer's point of view, it would be convenient to have an answer to the following problem: for a given choice of Z_{oo} and Z_{oe} find the corresponding bandwidth and the standing wave ratio of the dc block.

The universal graphical solution to this problem can be presented on a normalized (z_e, z_o) plane. It is assumed that the load and source impedances are equal to each

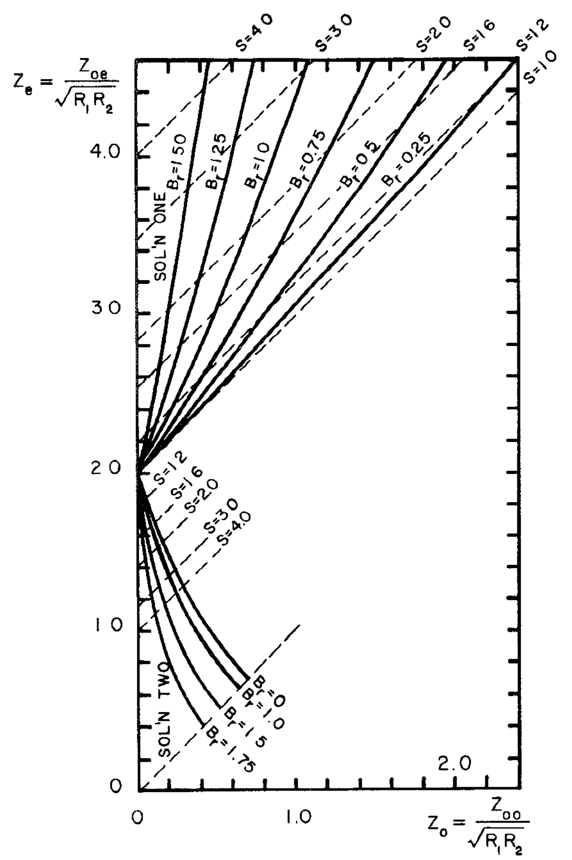

Fig. 5. Universal (z_e, z_o) plane.

other, so that the two solutions for the normalized even- and odd-mode impedances for the ripple behavior are specified by (23) to (26). For solution one, an auxiliary frequency function $\Phi(\Omega_c)$ can now be defined as follows:

$$\Phi(\Omega_c)=\frac{1+\sqrt{1+\Omega_c^2}}{\Omega_c^2}. \tag{40}$$

If Φ is eliminated from (23) and (24), the following is obtained:

$$z_e-z_o=2\sqrt{S}\,. \tag{41}$$

On the (z_e, z_o) plane, this is a straight line inclined 45° (if z_e and z_o are drawn in the same scale), which intersects the z_e axis at the point $2\sqrt{S}$. All the points located above the line $S=1$ belong to solution one.

The bandwidth can be found by eliminating S from (23) and (24). The result of such elimination is

$$z_e^2-\frac{2\Phi+4}{\Phi}\cdot z_ez_o+z_0^2=4. \tag{42}$$

This is the equation of a rotated hyperbola, centered at the origin, as shown in Fig. 5. For each relative bandwidth B_r, a corresponding Ω_c was computed by (16), then $\Phi(\Omega_c)$ was computed by (40), and the resulting curve $z_e(z_o)$ was plotted point by point. The obtained diagram is universal, since it is plotted in normalized values z_e and z_o. The designer can immediately spot in which direction on the (z_e, z_o) plane he must move in order to increase the bandwidth or to reduce the standing wave ratio.

If one desires a numerical answer instead of the graphical, one may use (41) directly for computation of the standing wave ratio. In order to compute the bandwidth, one solves (42) for Φ

$$\Phi=\frac{4z_ez_o}{(z_e-z_o)^2-4}. \tag{43}$$

Next, (40) is solved for Ω_c^2

$$\Omega_c^2=\frac{1}{\Phi}\left(\frac{1}{\Phi}+2\right). \tag{44}$$

Finally, one finds the relative bandwidth from (18)

$$B_r=2-\frac{4}{\pi}\cot^{-1}\Omega_c. \tag{45}$$

It may be seen from (44) that any positive values of Φ results in a real B_r. Therefore, all the points above the straight line for $S=1$ give an acceptable ripple solution with a real positive value for Ω_c.

The discussion of solution two runs similarly, yielding a somewhat more restricted area on the (z_e, z_o) plane. First, an auxiliary frequency function $\Psi(\Omega_c)$ is defined as follows:

$$\Psi(\Omega_c)=\frac{-1+\sqrt{1+\Omega_c^2}}{\Omega_c^2}. \tag{46}$$

When Ψ is eliminated from (25) and (26), the following is obtained:

$$z_e-z_o=\frac{2}{\sqrt{S}}. \tag{47}$$

This equation specifies a family of straight lines inclined 45° with respect to the z_o axis. These lines are located below the line $S=1$, as can be seen in Fig. 5. The bandwidth may be read from the curves which are obtained by eliminating S from (25) and (26)

$$z_e^2+\frac{4-2\Psi}{\Psi}z_ez_o+z_0^2=4. \tag{48}$$

These are again hyperbolas. When (48) is solved for Ψ, one obtains the expression for numerical determination of Ψ

$$\Psi(\Omega_c)=\frac{4z_ez_o}{4-(z_e-z_0)^2}. \tag{49}$$

The cutoff frequency variable is then

$$\Omega_c^2=\frac{1}{\Psi}\left(\frac{1}{\Psi}-2\right) \tag{50}$$

and the bandwidth B_r is afterwards computed by (45). The largest value which Ψ can achieve is 0.5, otherwise Ω_c^2 becomes negative and there is no real solution for the ripple bandwidth. In Fig. 5 is seen that the points belonging to solution two are located in the area below the $B_r=0$ curve. For common microstrip substrates and for the characteristic impedance 50 Ω, this area is usually not within the range of practical dimensions. For example, taking $S=1.2$ and $B_r=0.5$, the solution two for the 50 Ω impedance requires $Z_{oe}=93.4$ Ω and $Z_{oo}=2.14$ Ω. These values would require an extraordinarily narrow gap in the conventional microstrip technique. However, solution two may turn out to be feasible in another technique, such as the suspended microstrip.

Fig. 5 clearly identifies several regions on the (z_e, z_o) plane. The straight line inclined 45°, starting at the point $z_e=2$, $z_o=0$, designates the points which produce the maximally flat response. The rippled response for solution one (i.e., the response with $a_m=1$) is obtained if the point is selected above this straight line, whereas the points immediately below the straight line produce a nonrippled response. The area around the origin, limited by the curve $B_r=0$ also results in a rippled response, which corresponds to solution two (with $a_m>1$, the ripple not fully developed). Therefore, for any particular choice of Z_{oe} and Z_{oo} the designer may immediately see from the diagram which type of response will be obtained.

VI. Effect of the Dielectric Inhomogeneity

On a symmetrical coupled microstrip, the propagation modes are not pure transverse electromagnetic (TEM), but rather hybrid electromagnetic (HEM) [8]. The two dominant HEM modes, the odd and even modes, have no cutoff frequency and exhibit a behavior common to TEM modes; because of that, they are called quasi-TEM modes. While both odd and even TEM modes have equal wavelengths, the wavelength of the odd and even quasi-TEM modes on microstrip may differ from each other for 10 percent or more. In this section, it will be investigated how the difference in wavelengths influences the frequency response of the dc block.

In the low microwave region the dispersion is not significant, and both even- and odd-mode electrical lengths θ_e and θ_o of a coupled section of physical length l are linear functions of frequency as dictated by (2) and (3). The linear relationship is illustrated in Fig. 6. The average value of the two electrical lengths is denoted by θ_{av}

$$\theta_{av}=\frac{\theta_e+\theta_o}{2}. \tag{51}$$

The deviation of θ_e or θ_o from θ_{av} is denoted by δ

$$\delta=\theta_{av}-\theta_o=\theta_e-\theta_{av}. \tag{52}$$

In terms of the effective dielectric constants of the even and odd modes δ becomes

$$\delta=\frac{\pi f}{2f_0}\left[\frac{\sqrt{\epsilon_{re}}-\sqrt{\epsilon_{ro}}}{\sqrt{\epsilon_{re}}+\sqrt{\epsilon_{ro}}}\right]. \tag{53}$$

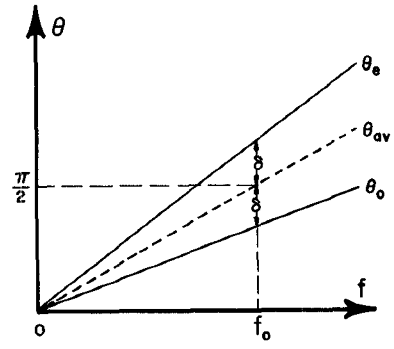

Fig. 6. Nondispersive behavior of θ_e and θ_o.

The computed values of ϵ_{re} and ϵ_{ro} for the microstrip substrate with $\epsilon_r=9$ are shown in Fig. 4(b). For a given choice of Z_{oe} and Z_{oo}, the normalized dimensions s/h and w/h may be read from Fig. 4(a). Then, the corresponding ϵ_{re} and ϵ_{ro} are found in Fig. 4(b). It is seen from the figure that ϵ_{re} and ϵ_{ro} do not differ much from each other. Therefore, angle δ is small in comparison with $\pi/2$.

When (52) is substituted in (7) the amplitude of the transmission coefficient S_{21} becomes

$$|S_{21}|^2=\frac{4(1+\tan^2\delta)(z_e-z_o)^2(1+\Omega_{av}^2)U^2}{V^2+W^2} \tag{54}$$

where the quantities U^2, V^2, and W^2 are defined as follows:

$$U^2=\left[\frac{z_e(1-\Omega_{av}\tan\delta)-z_o(1+\Omega_{av}+\tan\delta)}{(z_e-z_o)(1-\Omega_{av}^2\tan^2\delta)}\right]^2 \tag{55}$$

$$V^2=4\kappa^2\left[\frac{z_e(\Omega_{av}-\tan\delta)(1-\Omega_{av}\tan\delta)+z_o(\Omega_{av}+\tan\delta)(1+\Omega_{av}\tan\delta)}{1-\Omega_{av}^2\tan^2\delta}\right]^2 \tag{56}$$

$$W^2=\left[\frac{1}{2}(z_e^2+z_o^2+4)-z_ez_o\left(\frac{1+2\Omega_{av}^2+\tan^2\delta\Omega_{av}^2}{1-\Omega_{av}^2\tan^2\delta}\right)\right]^2. \tag{57}$$

Since δ is small, terms $\tan^2\delta$ can be neglected. Introducing again the coefficients a_0, a_2, and a_4, the frequency response simplifies to

$$|S_{21}|^2=\frac{1+\Omega_{av}^2}{a_0+a_2\Omega_{av}^2+a_4\Omega_{av}^4}\cdot\left(\frac{1-e_1}{1-e_2}\right). \tag{58}$$

The first part of the expression is identical with the frequency response of a pure TEM dc block, with Ω_{av} taking the place of the frequency variable. The part in parenthesis consists of two error terms specified as follows:

$$e_1=2\frac{z_e+z_o}{z_e-z_o}\Omega_{av}\tan\delta \tag{59}$$

$$e_2=2\kappa^2\frac{z_e+z_o}{z_e-z_o}\cdot\frac{1+\Omega_{av}^2}{a_0+a_2\Omega_{av}^2+a_4\Omega_{av}^4}\Omega_{av}\tan\delta. \tag{60}$$

Both these terms are small in comparison with unity and both have the same sign. When $e_1 = e_2$, the factor in parenthesis of (58) becomes unity, and $|S_{21}|^2$ is expressed by the same formula as in the pure TEM case. If the coefficients a_0 to a_4 are selected in accordance with solution one, it is seen from (60) that $e_2 = e_1$ at the two frequencies $\Omega_{av} = \Omega_m$ and $\Omega_{av} = -\Omega_m$. At these two frequencies the ripple response reaches unity for the quasi-TEM section, just the same as it did for the pure TEM section. At other frequencies there is a small difference between the two responses. This difference has been computed numerically for the design example from Section VII and it has been found that within the passband the difference from the ideal response is less than 0.002 dB.

The design of the microstrip dc block can be therefore based on the formulas which were derived previously for the pure TEM case. After selecting Z_{oe} and Z_{oo}, the values of the ϵ_{re} and ϵ_{ro} are found from the diagrams in Fig. 4(a) and 4(b). Then the "average" effective dielectric constant is computed from

$$\sqrt{\epsilon_{r,\mathrm{av}}} = \frac{\sqrt{\epsilon_{ro}} + \sqrt{\epsilon_{re}}}{2} \tag{61}$$

and the physical length of the coupled section is selected to be one quarter of the average wavelength

$$l = \frac{\lambda_{\mathrm{av}}}{4} = \frac{\lambda}{4\sqrt{\epsilon_{r,\mathrm{av}}}} \tag{62}$$

where λ is the free-space wavelength. The resulting ripple behavior is, for all practical purposes, identical with an ideal response of a pure TEM case.

VII. Design Example

The design objectives were chosen as $S = 1.19$ and $B_r = 0.4$ with $R_1 = R_2 = 50\ \Omega$. From (23) and (24), the normalized even and odd impedances (solution one) are computed to be $z_e = 3.300$ and $z_o = 1.119$. Denormalizing in accordance with (4) gives $Z_{oe} = 165.0\ \Omega$ and $Z_{oo} = 55.93\ \Omega$. The dc block is to be constructed on a substrate 1.857 mm thick (62.5 mil) with $\epsilon_r = 9.0$. From diagram 4(a) the normalized dimensions are found to be $w/h = 0.097$ and $s/h = 0.11$. Therefore, $w = 0.15$ mm and $s = 0.17$ mm. From diagram 4(b), the corresponding effective dielectric constants are found to be $\epsilon_{re} = 5.67$ and $\epsilon_{ro} = 5.02$. Then the average effective dielectric constant from (61) is $\sqrt{\epsilon_{r,\mathrm{av}}} = 2.31$. Taking the center frequency to be 3.0 GHz, the length of the coupled section is computed from (62) to be 10.82 mm.

The fabrication procedure by photoetching does not result in very precise dimensions. Under the microscope, it was found that the average conductor width is 0.158 mm and the average spacing is 0.227 mm, the latter being considerably off the design value. If these dimensions are used to recompute the actual values of the even- and odd-mode characteristic impedance, it is found that $Z_{oe} = 158.6$ and $Z_{oo} = 59.19\ \Omega$ (instead of 165.0 and 55.93). It would be possible to correct the mask accordingly and repeat the fabrication so that the dimensions come closer to the design values. This was not done, and the device was measured as fabricated.

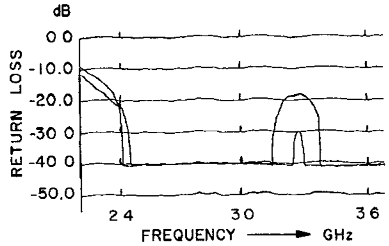

Fig. 7. Measured return loss.

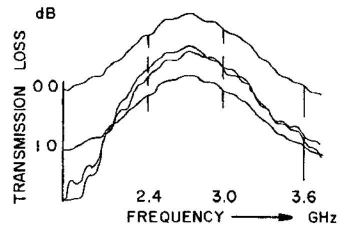

Fig. 8. Measured transmission loss.

The measured results are shown in Figs. 7 and 8. Each figure includes two separate recordings for the forward and reverse connections of the microstrip dc block, respectively. The input standing wave ratio in Fig. 7 is better than 1.19 (−21 dB) in the range from 2.4 GHz to about 3.6 GHz, with an exception at 3.36 GHz. Taking into account that the transitions from coax to microstrip may be producing some additional mismatch, the agreement for the input standing wave ratio is satisfactory.

The transmission loss is shown in Fig. 8. Within the design bandwidth, the measured value is found to vary between 0.6 dB and 0.8 dB. About 0.15 dB of this loss is attributed to the microstrip and coaxial line leading to the coupled section, and the remaining attenuation is due to the disspative loss within the coupled section.

It may come as a surprise that the ripple is not visible in the measured curves in Fig. 8. One must realize, however, that the ripple which corresponds to $S = 1.19$ is only 0.0328 dB, too small a value to be visible on the display utilized here. In addition to this, actual dimensions are not such as theoretical. In Fig. 4(a), the theoretical goal is point A, located on the straight line S-1.19. The actual dimensions of the coupled microstrip are presented by point B. It can be seen that we have actually achieved the maximally flat operation, since point B is located right on the line $S = 1$.

Another model designed for a narrower bandwidth and smaller standing wave ratio was also operating quite close to the design goals. The design procedure described above is thus found to be useful for practical applications, and the mechanical tolerances do not seem to be excessively demanding.

References

[1] D. LaCombe and J. Cohen, "Octave-band microstrip dc blocks," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 555–556, Aug. 1972.

[2] V. Rizzoli, "Analysis and design of microstrip DC blocks", *Micro-*

wave J., vol. 20, pp. 109–110, June 1977.
[3] G. I. Zysman and A. K. Johnson, "Coupled transmission line networks in an inhomogeneous dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 753–759, Oct. 1969.
[4] B. S. Vidula, "The synthesis of microstrip DC blocks," M. S. thesis, Univ. of Mississippi, University, MS, May 1979.
[5] B. Easter and B. S. Shivashankaran, "Some results on the edge-coupled microstrip section as an impedance transformer," *IEE J. Microwaves, Opt. and Acoust.* vol. 3, pp. 63–66, Mar. 1979.
[6] C. Y. Ho, "Analysis of dc blocks using coupled lines," *IEEE Trans. Microwave Theory Tech.* vol. MTT-23, pp. 773–773, 1975.
[7] D. R. Chambers, "Take the hassle out of microstrip coupling," *Microwaves*, vol. 14, pp. 48–50, July 1974.
[8] G. I. Zysman, D. Varon, "Wave propagation in microstrip transmission lines," in *Proc. G-MTT Int. Microwave Symp.*, (Dallas, TX) 1969, pp. 3–9.

Spectral Domain Analysis of Dominant and Higher Order Modes in Fin-Lines

LORENZ-PETER SCHMIDT AND TATSUO ITOH, SENIOR MEMBER, IEEE

***Abstract*—The spectral domain analysis is applied for deriving dispersion characteristics of dominant and higher order modes in fin-line structures. In addition to the propagation constant, the characteristic impedance is calculated based on the power-voltage definition. Numerical results are compared for different choices of basis functions and allow to estimate the accuracy of the solution.**

I. Introduction

The FIN-LINE structure is a special printed transmission line proposed for millimeter wave integrated circuits in 1973 by Meier [1]. Since then, a number of millimeter-wave components have been developed in the fin-line form (e.g., [2]). The single-mode range of frequency is relatively wide, as the fin-line somewhat resembles the ridged waveguide. Propagation characteristics of fin-line structures have been investigated by a number of workers such as Hofmann [3], Hoefer [4], [5], and Saad and Begemann [10]. In [3], which is based on Galerkin's method in the space domain, sinusoidal functions are used as expansion functions. Hence a comparatively large number of expansion functions is required to obtain accurate results and, in addition, relative convergence problems occur and have to be handled carefully. On the other hand, some engineering approximations are involved in the work in [4].

In the present paper, fin-line structures are analyzed using the spectral domain technique, which has been developed for the analysis of various printed transmission lines for microwave integrated circuits [6], [7]. In this method, the information on the propagation constant at a given frequency is extracted from algebraic equations that relate Fourier transforms of the currents on the fins to those of the electric field in the dielectric–air interface. These equations are *discrete* Fourier transforms[1] of coupled integral equations one would obtain if the formulation is done in the space domain. Obviously, algebraic equations are much easier to handle in numerical processing. In addition to standard features of the spectral domain technique, the present work contains the following provisions.

1) The accuracy of the method is checked by comparing results obtained from three different choices of basis functions. A convergence check is also performed by increasing the number of basis functions for one of these sets.

2) In addition, dispersion curves for higher order modes are presented. For practical applications, the knowledge of higher order modes is important, because often a single mode operation is required.

3) Another important quantity for design purposes is the characteristic impedance of the dominant mode. By applying a definition suitable to fin-line structures, useful results for the characteristic impedance could be obtained and are presented in this paper.

II. Formulation of the Eigenvalue Problem

Since the details of the spectral domain method itself have been reported in [6] and [7], only the key steps will be given here. The method of using alternative sets of basis functions for accuracy checks has recently been

Manuscript received March 17, 1980; revised April 15, 1980. This work was supported in part by the U.S. Army under Grant DAAG29-78-G-0145.

L.-P. Schmidt is with AEG-Telefunken N1 E32, Ulm, West Germany, on leave from the Department of Electrical Engineering, The University of Texas at Austin, Austin, TX 78712.
T. Itoh is with the Department of Electrical Engineering, The University of Texas at Austin, Austin, TX 78712.

[1]Henceforth referred to as Fourier transform.

0018-9480/80/0900-0981$00.75 ©1980 IEEE